2024 ■ Vol. 20 ■ no. 2

Exploring dynamic structures of dyadic conversations using categorical cross recurrence quantification analysis: A tutorial

Shirley Duong [a], Tehran J. Davis [b], Heather J. Bachman [a], Elizabeth Votruba-Drzal [a] & Melissa E. Libertus [a]

[a]University of Pittsburgh, Learning Research and Development Center
[b]University of Cincinnati, Center for Cognition, Action, and Perception, Department of Psychology

Abstract ■ Social interactions are defined by the dynamic and reciprocal exchange of information in a process referred to as mutual alignment. Statistical methods for characterizing alignment between two interacting partners are emerging. In general, they exploit the temporal organization of dyadic interactions to uncover the effect of one partner on the other and the extent to which partners are aligned. This paper describes and provides a tutorial on one such method, categorical cross recurrence quantification analysis (CRQA), which quantifies the temporal structure and co-visitation of individual and sequential states of interest. CRQA is a useful descriptive technique that can be used to explore the extent, structures, and patterns of partner alignment within dyadic interactions. We provide a brief technical introduction to CRQA and a tutorial on its application to understanding parent-child linguistic interactions using the 'crqa' package in R (Coco, Monster, Leonardi, Dale, & Wallot, 2021).

Keywords ■ Dyadic interactions, categorical data, cross recurrence quantification. **Tools** ■ R.

shd77@pitt.edu

10.20982/tqmp.20.2.p121

Acting Editor ■ Denis Cousineau (Université d'Ottawa)

Reviewers ■ One anonymous reviewer.

Introduction

There is considerable interest in the psychological sciences to uncover more sensitive measures of dyadic interactions, such as how partners co-construct actions and conversation, the extent to which partners reciprocate or adapt to each other's input, and the temporal organization of these interactions (e.g., Bollenrücher et al., 2023; Gallotti et al., 2017; Solomon et al., 2021). For instance, many child development researchers aim to understand how linguistic interactions between adults and children may contribute to the acquisition of a variety of skills, such as language (e.g., Anderson et al., 2021) and math (e.g., Duong et al., 2021; Fox et al., 2024). A typical analysis would involve examining the type and quantity of language input by one or both speakers in naturalistic and structured settings and their associations with children's outcomes.[1] To achieve this, rich observational data are often condensed or collapsed into aggregate measures of talk, such as frequencies of words, utterances, or exchanges, and correlations are computed between these frequency variables and the developmental outcomes of interest.

Researchers often suggest that the links between the frequency of input and children's skills are partially driven by (1) the alignment, reciprocity, or exchange of information between speakers and (2) specific types of language input that elicit certain structures of talk (e.g., after one speaker prompts the other with more cognitively demanding input, the dyad engages in longer exchanges). However, these conclusions are rarely formally tested or explored with appropriate quantitative methods (e.g., lag sequential analysis, autoregressive models, longitudinal action-

[1]To clarify, these studies typically use counts of utterances or words from individual speakers, rather than their co-occurrence. However, we acknowledge a large body of work on synchrony in infant-mother interactions that consider the frequency of co-occurring behaviors (see Leclère et al., 2014, for a systematic review), which contrasts with the point being made here.

Table 1 ■ Example of a parent-child interaction for illustrating mutual alignment

Order	Speaker	Utterance
1	Parent (P)	Okay, can I have chocolate please?
2	Child (C)	A two or one?
3	P	Two.
4	P	How much are two?
5	C	One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen.
6	P	Thirteen?
7	P	Hmm.
8	C	You don't have enough.
9	P	Or thirteen cents?
10	C	Thirteen cents.
11	P	Okay, so I have a ten and a ten so that's twenty.
12	P	That's twenty cents.
13	C	You don't have enough.
14	P	So, if I give you my twenty cents, you have to give me- that's thirteen.
15	P	You have to give me seven pennies in there that you can count out.

Note. "Order" refers to the order in which the utterance is spoken in time.

partner interdependence models). In this paper, we describe and provide a simple tutorial to cross recurrence quantification analysis (CRQA), a technique that can quantify partners' alignment in social interactions and the nature of such alignment. Researchers can apply CRQA to existing, temporally ordered observational data (e.g., parent-child conversations during play, teacher-student interactions in the classroom) to gain a deeper understanding of dyadic conversations that align with their hypothesized underpinnings of social interactions.

Theoretical motivation

The application of CRQA to the study of dyadic conversations aligns with the view that the defining features of social interactions are the dynamic and reciprocal exchange of information, thoughts, actions, and experiences, rather than partners' shared goal or activity (Gallotti et al., 2017). Conversations between two partners are adaptive, complex, iterative, non-stationary, self-organizing, and sensitive to feedback (Cox & van Dijk, 2013; Menninga et al., 2017; van Dijk et al., 2013). Subsequent conversational inputs such as words or utterances are products of a transactional process: One speaker adapts their linguistic output to the other's earlier (patterns of) input, as well as global factors such as their perception of the other's linguistic abilities (Cox & van Dijk, 2013; Dale & Spivey, 2005, 2006; Denby & Yurovsky, 2019; Fusaroli et al., 2023; Misiek et al., 2020; Yurovsky et al., 2016). This process whereby partners reciprocally exchange information and adjust their input in varying degrees and patterns is referred to as mutual alignment. Defining social interactions in terms of the alignment between partners implies that we can better understand dyadic interactions when they are examined at the level of the group rather than the individual (Gallotti et al., 2017). Moving forward, the term "alignment" will be used to refer to mutual or partner alignment in this paper.

What does alignment look like in conversations? Table 1 shows a toy example of a parent-child interaction comprised of fifteen utterances. This example is derived from data used in the companion paper to this tutorial, where we describe an application of CRQA to examine the associations between patterns in parent-child conversations about number concepts and children's math abilities (Duong, Davis, et al., 2024). In this toy example, the dyad is engaging in pretend grocery play, where the parent assumes the role of a customer and the child plays a store associate who rings up the parent's purchases. Suppose we are interested in speakers' use of utterances containing number words, i.e., number utterances (e.g., Utterance #2, "A two or one?") because they offer children an opportunity to learn about and display their understanding of number concepts. Specifically, we would like to know when one speaker's number utterance is followed by one or more number utterances by the other speaker, which we assume reflects their joint focus on numerical concepts. We can describe this pattern of conversation as the alignment of partners' number utterances. For example, in this conversation, the parent says, "How much are two?" (Utterance #4) when referring to the cost of chocolates, and the child responds by counting, "One, two, three, . . . , thirteen" (Utterance #5). The child's response is then followed by the parent's confirming question, "Thirteen?" (Utterance #6). This reciprocal exchange of information about the cost of a grocery item is one illustration of the alignment of parent-child utterances about a specific topic, in this case, talk about numbers. Given conversations comprised of many more utterances, we can describe or quantify alignment by applying CRQA.

Cross recurrence quantification analysis (CRQA)

Recurrence quantification analysis (RQA) and its bivariate extension, cross-recurrence quantification analysis (CRQA), is a non-linear method of data analysis that allows researchers to visualize and quantify the frequency and duration of repeated events or states (i.e., recurrences) within dynamic systems (Angus, 2019; Coco et al., 2021; Coco & Dale, 2014; Davis, 2017; Eckmann et al., 1987; Fusaroli et al., 2014; Rohlfing et al., 2020; Solomon et al., 2021; Wallot & Leonardi, 2018; "Recurrence Plots and Their Quantifications: Expanding Horizons," 2016; Webber & Zbilut, 1994, 2007; Zbilut & Webber, 1992). CRQA is used to examine the behavior of two interacting systems and thus, it can be employed to understand dyadic conversations. The interdependence between speakers is illustrated with recurrence plots and metrics reflecting the extent to which speakers are aligned, as well as their patterns of alignment, e.g., whether speakers are engaging in back-and-forth exchanges of information. Recurrence analysis has been used extensively in fields that deal with time series data and alignment within and between targets, such as verbal and motor communicative behaviors (Lira-Palma et al., 2018; Xu & Yu, 2016), eye movements and other motor behaviors (Afsar et al., 2018; Richardson & Dale, 2005), physiology and health (Curtin et al., 2017; Heunis et al., 2018), emotions (Main et al., 2016), decision-making (McCormick & Blaha, 2021), and economics (He et al., 2020).

Several methods can be applied to studying alignment within dyadic interactions including lag sequential analysis (e.g., Bakeman & Quera, 2011), autoregressive models, and longitudinal action-partner interdependence models (LAPIM; e.g., Bollenrücher et al., 2023; for more methods, see Gates & Liu, 2016). In general, these methods can be used to uncover the temporal evolution of states and provide, at a global level, the effect of one partner on the other and the extent of partner alignment. For instance, autoregressive models (Chen & Ferrer, 2022; Hamaker et al., 2009) and LAPIM (Bollenrücher et al., 2023) can be used to examine how one partner's behavior at a particular time point influences the other's behavior at the same or another time point (referred to as "lags"). These methods allow researchers to capture the effect of partners' behaviors over time and characterize types of dyadic relationships (Bollenrücher et al., 2023; Chen & Ferrer, 2022; Hamaker et al., 2009). Lag sequential analysis can be used to assess what states tend to follow or precede specific states.

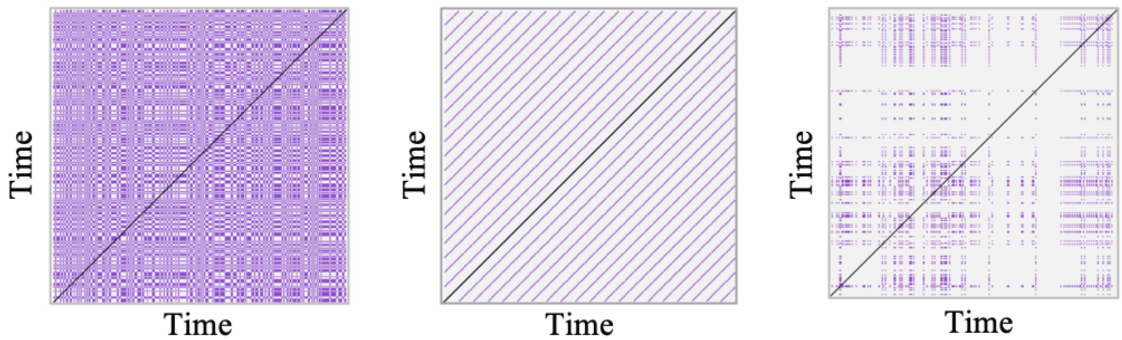
However, with these methods, it is complicated to consider many lags at once or all possible lags, which can be useful for tracking how alignment changes over time. Moreover, these models do not describe the temporal patterns or durations of partner alignment and thus, they may not provide a comprehensive view of dyads' behaviors. CRQA addresses this limitation by calculating measures that describe patterns of alignment (e.g., Dale et al., 2011). Note that unlike the methods described above, CRQA is *descriptive*, i.e., statistical inferences cannot be drawn from the recurrence measures alone. CRQA provides a systematic way to identify patterns of interaction and given many dyads, researchers can use other statistical techniques to make inferences about different kinds of or trends in dyadic interaction patterns. For instance, one may be interested in understanding what interaction patterns are linked with parental sensitivity or warmth and child responsiveness. In this case, CRQA could be used to uncover the types and durations of behaviors and practices that describe quality parent-child interactions and inform early intervention programs.

In this article, we first describe the steps of executing CRQA and then illustrate an application of CRQA to parent-child linguistic interactions during pretend play, with examples in the R programming language. Additionally, we show what CRQA offers over traditional count variables, such as the number of utterances by each speaker, to our understanding of dyadic interactions.

Categorical CRQA

While CRQA can be applied to continuous data (see Coco & Dale, 2014), we focus our paper on categorical data, given that the states of interest in dyadic conversations tend to be categorical in nature, e.g., types of syllables, babbles, words, parts of speech, utterances, and conversational turns. Beyond the linguistic context, other interesting categories or types of states in observational data, including gestures and looks (e.g., a person's smile or gaze at a particular object), and survey data, such as attitudes and feelings over time (e.g., sad or happy in a specific context or at a particular point in time), are also categorical. In this tutorial, we apply categorical CRQA to utterance-level data. CRQA requires no assumptions about the data. In the categorical case, the data are encoded into integers for each unique unit of interest. Specifically, each utterance is assigned an integer based on the state(s) of interest. In the toy example (Table 1), if we were interested in the occurrence of number utterances, one way to encode the data is to assign each utterance a value of 0 = *non-number utterance* or 1 = *number utterance*. The data can also be encoded to reflect who is speaking at the time, e.g., in the toy example, non-number talk from the parent and non-number talk from the child would be assigned different numerical values.

Figure 1 ■ Example recurrence plots. Typology of recurrence plots: (left) uniformly distributed noise, (middle) periodic system, (right) naturalistic, disrupted system. The recurrence plots representing conversations between two speakers are more likely to look like the plot on the right than the other plots.



Hyperparameter selection

The first step of performing CRQA is to identify hyperparameters that control how recurrence (or conceptually, alignment) is calculated. At a minimum, most common CRQA packages require the user to specify three key hyperparameters: embedding dimension, delay, and radius. For categorical data, we only need to specify the radius, which sets a threshold for determining whether the distance between two states indicates recurrence. Given that we encode the data numerically, the radius represents the maximum allowed distance between states in the category space for them to be considered recurrent. Returning to our toy example, the distance between non-number utterances (encoded as 0) and number utterances (encoded as 1) is 1. Setting the radius parameter to a value less than 1 means that in order for states to be considered recurrent, they must be less than 1 unit apart. In the case of categorical data that are encoded as integers, this ensures that recurrence is only indicated for exact matches. Typically, when dealing with categorical data we set this value to near zero (e.g., 0.01) to highlight that we are only interested in exact matches—a state is considered recurrent with itself (1 matches with 1 or number utterances match with other number utterances, and 0 matches with 0 or non-number utterances match with other non-number utterances), but not with another state (1 does not match with 0 or number utterances do not match with non-number utterances).

The other hyperparameters, embedding dimension and delay, are not particular to CRQA itself, but instead define optimal parameters for the phase space reconstruction. In brief, phase space reconstruction involves creating a multidimensional representation of a dynamical system by using time-delayed copies of a single dimension—the original time series (Takens, 1981). This method allows measures of a single dimension to act as surrogates for other, unmeasured dimensions. While crucial for CRQA involving continuous time series (e.g., examining movement coordination), phase space reconstruction is not typically used with categorical data. Continuous data require phase space reconstruction to capture high-dimensional dynamics, whereas categorical data, like linguistic exchanges, are often lower-dimensional or discrete. The dynamics of categorical phenomena are best captured in sequences of discrete states, not in a continuous phase space. Therefore, considerations about parameterizing delay and embedding dimension are not usually applicable to categorical data.

This tutorial focuses on recurrence patterns within the sequence of categorical states, using the radius parameter to define the similarity threshold for recurrences. Thus, we capture the system's dynamics through transitions between discrete states without needing phase space reconstruction. In packages that require a selection of delay and embedding dimension, a delay of 0 (no delay) and embedding dimension of 1 (using on the original timeseries) are used for categorical CRQA (Dale & Spivey, 2006; Dale et al., 2011). For those interested, Wallot and Leonardi (2018) offer an excellent introduction to parameter estimation for phase space reconstruction in non-categorical CRQA.

Recurrence plots (RPs)

In the context of dyadic conversations, streams of talk can be thought of as two systems, and the shared trajectory of these two data series are conceptualized as when the speakers visit the same states over time. The overlap between these two data series is visualized with a recurrence plot (RP), a (sparse) square matrix that indicates where a state recurs across all possible moments in time between two dynamical systems. Specifically, we have a sequence of m states or codes for one speaker, $P = (p_1, p_2, \ldots, p_m)$, and a sequence of n states or codes for the other speaker, $C = (c_1, c_2, \ldots, c_n)$, and these are plotted against each other with P on the x-axis and C on the y-axis. This creates an $m \times n$ matrix, where $m = n$, in which each element represents a pair of states (i, j), where i is the ith element of P and j is the jth element of C.

Cross recurrence refers to indices in the matrix where i and j are equivalent; these elements in the matrix are assigned a value of 1 and all other, non-recurrent pairs of states are given a value of 0. Visual representations of RPs typically look like color grids, where elements corresponding to co-occurring states are colored while the rest of the plot tends to be white or grey. One key feature of cross-recurrence plots is the line of incidence (LOI). The LOI is the major diagonal that reflects moments in time when $P_m = C_n$. In natural exchanges, instances of recurrence along the LOI are atypical, but if they do occur, they reflect moments where both speakers performed the state of interest at the same moment in time (e.g., Coco & Dale, 2014; Davis, 2017; Solomon et al., 2021), such as both partners speaking a number utterance. Although many researchers have examined recurrence at certain distances away from the LOI (i.e., measured the alignment of speakers in a subsection of the plot) or above and below the LOI (i.e., determined whether one speaker is leading or following some coordination), this paper will focus on "global" recurrence measures, which quantify the entire plot.

As a whole, the topology of the RPs depict how states are organized across time points (see Figure 1). For instance, homogeneous RPs, which look like uniformly distributed noise, describe systems that are stationary or stay in the same state for the majority or all of the time. Periodic RPs, which can look like checkerboards or rugs that contain the same repeating pattern, are characteristic of systems that switch between states of interest in a cyclical manner. RPs with disruptions or inconsistencies can point to where states are concentrated and thus when they recur in time. Specifically, a visual scan of dyadic interactions' RPs can reveal the consistency of states and when states recur most or least frequently.

Quantification of RPs

Several descriptive measures or parameters describing the RP, and thus corresponding to the behavior of the dyad, can be derived using CRQA, including (but not limited to) the extent of dyads' alignment in speech and the lengths of exchanges and consecutive conversational inputs. This tutorial focuses on extracting the following commonly used measures: (1) recurrence rate, (2) determinism, (3) mean diagonal line, (4) laminarity, and (5) trapping time.

Recurrence rate (RR) is often used to capture dyads' overall alignment or reciprocity during the conversation, which is the extent to which a speaker's linguistic input changes to accommodate the behaviors or speech of the other person. This may look like speakers' inputs mirroring each other, e.g., based on the words that speakers use, the types of utterances spoken or concepts discussed, or the syntactic structure of sentences. Specifically, RR is the percentage of recurrence points in an RP, i.e., the number of elements (i, j) where $i = j$ divided by the area of the RP ($m \times n$, where $m = n$).

Determinism (DET) captures the proportion of recurrent states that occur in extended sequences and broadly reflects the degree to which states follow predictable patterns. In the context of dyadic alignment at, for instance, the level of utterances, DET reflects the extent to which dyads exhibit "back and forth" exchanges, such that one speaker utters speech and this is followed by an utterance of the same kind by the other speaker. Visually, these sequences appear as connected, diagonal lines of recurrence on the RP, and may be contrasted against moments of recurrence that do not occur as part of a sequence, e.g., moments of talk by either speaker that are not immediately reciprocated by the other. DET is calculated by taking the ratio of recurrent points that make up diagonal lines of a given length d (where $d > 2$) on the RP against the total number of recurrent points. At the level of utterances, a length of two captures the shortest sequence of exchanges, e.g., an utterance from one speaker and a response from the other. Relatedly, the *mean diagonal line* (MeanL) refers to the average length of all diagonal lines on the RP, e.g., the duration or length of "back and forth" exchanges.

Laminarity (LAM) describes the extent to which one speaker or the dyad continuously visits a state, e.g., repeatedly employs the same type of utterance after the other speaker does. Thus, at the level of utterances, LAM captures instances in which one speaker initiates an exchange, but rather than a "back and forth", the second speaker responds with several successive utterances. LAM is computed as the ratio of recurrence points that form vertical lines of length l (again, typically $l > 2$) compared to the total number of recurrent points. Finally, *trapping time* (TT) is calculated by taking the average length of all vertical lines, thus reflecting the average duration of laminar states.

An illustration of categorical CRQA

In this section, we provide a simple tutorial on an application of CRQA to transcription data of parent-child dyads engaged in pretend play. Specifically, these transcriptions were annotated for instances of talk about numbers at the utterance-level and CRQA was applied to examine the reciprocal exchange of number utterances, hereinafter referred to as "number talk," between speakers. A detailed description of these interaction data is provided below. Additionally, we show that the metrics obtained from CRQA can offer something beyond traditional talk measures, e.g., the count of utterances. To do so, we derive the RPs and recurrence metrics for dyadic conversations with similar frequencies of number talk or alignment (e.g., recurrence rates) and variation in their temporal organization. The full code for this tutorial is available at https://github.com/s-duong/crqa-number-talk.

Required R package

We demonstrate the application of CRQA using functions from the `crqa` package in R (Coco & Dale, 2014; Coco et al., 2021). This package contains different methods for computing recurrences, including a core recurrence function, `crqa()`, which takes two time or state series (in the form of vectors) and specified hyperparameters (i.e., radius, delay, and embedding dimension) to calculate an RP and different metrics (e.g., recurrence rate (RR), determinism (DET), laminarity (LAM)). Additionally, we use `ggplot2` (Wickham, 2016) to create more visually appealing RPs from the recurrence matrix provided in the output of the `crqa()` function.

Data source

The transcription data for this tutorial were derived from the Parents Promoting Early Learning Study, a community-based, longitudinal study examining socioeconomic variability in the home learning environment and four-year-old children's early academic skills (for papers published with these data, see, e.g., Bachman et al., 2020, 2022; Fox et al., 2024; Duong et al., 2021). Parents and children engaged in a pretend grocery shopping activity with a set of developmentally appropriate toys, including pretend food items, fake bills and coins, and a cash register; they were instructed to play with the materials as they normally would for about 8 minutes. Each parent-child interaction was video recorded and transcribed at the utterance level (Pan et al., 2004) by trained research assistants using Datavyu, an open-source behavioral coding software (Datavyu Team, 2014).

Next, research assistants reviewed each transcript and annotated instances of number talk (based on coding schemes by various authors, e.g., Bachman et al., 2020; Ramani et al., 2015). Number talk utterances included discussions of arithmetic, patterns, comparing magnitudes, ordinal relations, counting, identifying number symbols, labeling sets of objects, and other number or math concepts (e.g., time). The coders searched and evaluated key terms, such as number words (e.g., "four"), ordinal words (e.g., "third"), elicitations (e.g., "how many"), and terms associated with specific math concepts (e.g., "count," "add," "take away"). Utterances containing the key terms not used in a math context were not coded as number talk. For instance, utterances that used "one" to refer to an object (e.g., "You want that one?") rather than the quantity (e.g. "I have one egg in the basket") or "count" to refer to something other than enumeration (e.g., "That time didn't count and I want to go again") were not considered number talk.

Conversations from five dyads were chosen as examples for this tutorial. The toy example presented earlier (Table 1) will be used to show the application and output of the `crqa()` function and RP creation using `ggplot2`. The purpose of applying CRQA to a small subset of one dyad's conversation is to clearly show how streams of state codes map onto the transcription, are transformed into a recurrence matrix, and then quantified. Next, RPs and metrics will be derived for four additional dyads. Two dyads (Dyads 1 and 2) have similar frequencies of number talk and different RR, DET, and LAM. The other two dyads (Dyads 3 and 4) have similar RRs, but different DET and LAM (see Table 3 for interpretations of these different metrics). The code to obtain measures for these four dyads is identical to what is shown below for the toy example and can be found in the full tutorial code.

Data set-up

CRQA requires two vectors of values; our example includes one column representing parent talk and the other child talk with states (i.e., when each person spoke or listened) in their temporal order. Each vector contains strings or numbers representing the states of interest. In our case, each utterance was assigned one of these six state codes: (1) number talk by either speaker, (2) no utterance by either speaker during number talk of the other speaker, (3) parent non-number talk, (4) child non-number talk, (5) no utterance by the parent during non-number talk of the child, or (6) no utterance by the child during non-number talk of the parent. The parent-specific codes, i.e., non-number talk and no utterance by the parent during non-number talk of the child, only appeared in the parent talk column (and vice versa for the child codes). Since the primary state of interest was number talk and CRQA is concerned with when states co-occur, we forced non-recurrence for other types of talk. Table 4 shows how the utterance-level data are coded for the toy example presented in Table 1.

Obtaining the recurrence metrics

To begin, hyperparameters were specified in the `crqa()` function, namely the radius, delay, and embedding dimension. Since we are working with categorial data, the radius is set to be near zero so that recurrence is only indicated using a match or mismatch of each state in time. Provided our coding scheme, we achieved this by setting our

Table 2 ■ Data set-up of the toy example (Table 1) for CRQA in R

Speaker	Utterance	Number talk?	P data series	C data series
Parent (P)	Okay, can I have chocolate please?	No	3	6
Child (C)	A two or one?	Yes	2	1
P	Two.	Yes	1	2
P	How much are two?	Yes	1	2
C	One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen.	Yes	2	1
P	Thirteen?	Yes	1	2
P	Hmm.	No	3	6
C	You don't have enough.	No	5	4
P	Or thirteen cents?	Yes	1	2
C	Thirteen cents.	Yes	2	1
P	Okay, so I have a ten and a ten so that's twenty.	Yes	1	2
P	That's twenty cents.	Yes	1	2
C	You don't have enough.	No	5	4
P	So, if I give you my twenty cents, you have to give me- that's thirteen.	Yes	1	2
P	You have to give me seven pennies in there that you can count out.	Yes	1	2

Note. The last two columns, 'P data series' and 'C data series', were created from the 'Speaker' and 'Utterance' variables from the original transcript and used for CRQA. The numerical codes in the data series have the following values: 1 = number talk (NT) by either speaker, 2 = no utterance by either speaker during NT of the other speaker, 3 = parent non-NT, 4 = child non-NT, 5 = no utterance by the parent during non-NT of the child, and 6 = no utterance by the child during non-NT of the parent.

radius to 0.5 (e.g., given that "number talk" = 1, other instances of "number talk" are within radius < 0.5, but "no utterance" = 2 is not). Additionally, a delay of 0 and embedded dimension of 1 were set, consistent with past applications of categorical CRQA (Dale & Spivey, 2006; Dale et al., 2011). Other arguments that we specified for this function are `mindiagline` and `minvertline`, which represent the minimum diagonal and vertical lengths of recurrent points, respectively. These are both set to 2, which mean that the minimum length of recurrent patterns is two utterances; this length captures the shortest possible length of an exchange, where one person uses number talk and next person responds with number talk. Listing 1 shows the parameters that were set for CRQA. With many dyads, it is useful to set the hyperparameters at the start of one's program if they will be repeated many times, as this reduces the chance of making errors.

Listing 2 shows how to obtain the recurrence measures from the toy example and Table 3 provides a brief description and interpretation of the output from this function, which includes the values RR, DET, MeanL, LAM, TT, as well as a sparse matrix of the RP. Other metrics that appear in the output that are not discussed here can be found in the package documentation (cran.r-project.org/web/packages/crqa/index.html). Each output parameter is an object that can be accessed using the basic extraction operator, $ (e.g., `example_crqa$RP` to obtain the RP).

In the toy example conversation, the extent to which number talk co-occurs is represented by the RR (21.33%). The interpretation of this value partially depends on the way the data were encoded. In this tutorial, we forced non-number talk to be non-recurrent and assigned parent- and child-specific codes to each utterance. Thus, even if both speakers used (or listened to) number talk for the entire conversation, the highest RR they could achieve is 50%. It is not accurate to say that the RR, in this example, represents the amount of time or quantity of utterances in which speakers showed alignment in number talk. Rather, the RR is a standardized metric of number talk alignment that is meaningful for comparing two or more conversations.

The remaining CRQA metrics describe the patterns of (number talk) alignment. In the toy example, fifty percent of the recurrent points make up diagonal lines (DET) and the mean length of those diagonal lines is two. One can interpret this to mean that dyads' co-occurring number talk is characterized by "back and forth" exchanges about half of the time and these exchanges are, on average, two utterances long. Thus, the typical pattern of alignment in the toy example is that one speaker uses a number talk utter-

Listing 1 ■ Hyperparameter settings for CRQA and data sequences for the toy example (Table 1)

```
radius_value = .5
delay_value = 0
embedding_value = 1
min_diag = 2
min_vert = 2

toy_parent <- c(3, 2, 1, 1, 2, 1, 3, 5, 1, 2, 1, 1, 5, 1, 1)
toy_child <- c(6, 1, 2, 2, 1, 2, 6, 4, 2, 1, 2, 2, 4, 2, 2)
```

Table 3 ■ Summary of recurrence metrics, their derivations, and interpretation

Recurrence metric	Derivation	Interpretation
Recurrence rate (RR)	Percentage of recurrent points on a recurrence plot (RP)	What is the extent to which dyads' number talk co-occurs? What is the extent of dyads' alignment in number talk?
Determinism (DET)	Percentage of recurrent points that make up diagonal lines of d length at minimum on a RP	What is the extent to which dyads engage in "back and forth" exchanges of number talk?
Mean diagonal line (MeanL)	Mean length of diagonal lines of d length	What is the average quantity of utterances that make up dyads' number talk exchanges?
Laminarity (LAM)	Percentage of recurrent points that make up vertical (and horizontal, in our case) lines of l length at minimum on a RP	What is the extent to which one speaker in the dyad employs consecutive number talk utterances after the other uses number talk?
Trapping time (TT)	Mean length of vertical (and horizontal, in our case) lines of l length	What is the average quantity of utterances that make up consecutive number talk utterances?

Note. The interpretations of the recurrence measures are stated in the context of the data used in this tutorial, i.e., parent-child number talk.

ance and the other responds with a number talk utterance. Moreover, 37.5% of the recurrent points make up vertical or horizontal lines (LAM), which can be interpreted as the extent to which either speaker is using consecutive number talk utterances after the other uses number talk. In other words, a little over one-third of all co-occurring number talk is characterized by one speaker's continued use of number talk utterances immediately after (e.g., as a response to) the other speaker's number talk. On average, the length of this consecutive number talk is two utterances.

Obtaining the recurrence plots (RPs)

We show two ways to obtain the RP, first using the `crqa()` function and then using `ggplot()`. The sparse matrix that is obtained from running the `crqa()` function can be accessed with `$RP` (e.g., run `example_crqa$RP` to obtain the toy example's RP). Unfortunately, this matrix can become very large and difficult to read if there are many time points to analyze (see Section 6.1 in the Supplemental Material). Thus, another way of visualizing the recurrence between two systems is to create a square raster plot using functions from the `ggplot2` and `reshape2` packages (Wickham, 2007, 2016) which standardize the size of the RP. We recognize that there are many ways to generate RPs and this is just one example.[2]

Listing 3 shows how the RP generated from `crqa()` on the toy example is used to create a more visually appealing RP. First, the sparse matrix (`$RP`) is back-transformed into three vectors of information using the `melt()` function from `reshape2`: the parent data series, the child data series, and a sequence of values indicating whether the events in each series co-occur (TRUE/FALSE). Every pairwise combination of codes from the two data series is assigned a logical value; if the data series have a length of t, then the `melt()` function creates vectors of t by t length. Then, this information is used as input for the `geom_raster()` function from the `ggplot2` library. `geom_raster()` generates a colored heat map, given

[2] `ggplot` may be relatively slow when representing large matrices, which becomes relevant when processing many files. The `SparseM` package in R (Koenker & Ng, 2003) has functions for creating sparse matrices that run faster, but are less visually appealing.

Listing 2 ■ Running CRQA with the toy example

```
# run crqa
example_crqa <- crqa(toy_parent,
                     toy_child,
                     radius = radius_value,
                     delay = delay_value,
                     embed = embedding_value,
                     mindiagline = min_diag,
                     minvertline = min_vert
                     )

# show output
example_crqa
## $RR
## [1] 21.33333
##
## $DET
## [1] 50
##
## $NRLINE
## [1] 12
##
## $maxL
## [1] 2
##
## $L
## [1] 2
##
## $ENTR
## [1] 0
##
## $rENTR
## [1] NaN
##
## $LAM
## [1] 37.5
##
## $TT
## [1] 2
##
## $catH
## [1] NA
##
## $RP
## 15 x 15 sparse Matrix of class "ngCMatrix"
##
##  [1,] . . . . . . . . . . . . . . .
##  [2,] . . | | . | . . | . | | . | |
##  [3,] . | . . | . . . . | . . . . .
##  [4,] . | . . | . . . . | . . . . .
##  [5,] . . | | . | . . | . | | . | |
##  [6,] . | . . | . . . . | . . . . .
##  [7,] . . . . . . . . . . . . . . .
##  [8,] . . . . . . . . . . . . . . .
##  [9,] . | . . | . . . . | . . . . .
## [10,] . . | | . | . . | . | | . | |
## [11,] . | . . | . . . . | . . . . .
## [12,] . | . . | . . . . | . . . . .
## [13,] . . . . . . . . . . . . . . .
## [14,] . | . . | . . . . | . . . . .
## [15,] . | . . | . . . . | . . . . .
```

x- and y-coordinates (i.e., the parent and child data sequences) and another variable that maps onto a color (i.e., `TRUE` or `FALSE` for the co-occurrence of number talk). Last, rugs are created using the `geom_rug()` function, which is typically used to supplement 2-dimensional plots by visualizing 1-dimensional marginal distributions. In our case, we use the rugs to show the occurrence of number talk in temporal order by individual speakers on the x- (parent) and y- (child) axes. These rug marks can be tallied to derive the frequency of number talk utterances per speaker.

The resulting RP has recurrent points colored in purple and the x- and y-axes are marked for when the state of interest occurs in temporal order. In this tutorial, the blue rug marks represent parent states and the red rug marks represent child states. One could change these colors by changing the arguments in the `scale_manual_color()` lines. Last, the black diagonal line is the LOI, which reflects when states occur at the same moment in time. In our case, recurrence along this line is not present because individuals in the dyad took turns speaking. A function for the construction of RPs given a dataset with multiple interactions is presented in the full code for this paper.

Figure 2 shows the resulting plot for the toy example. Note that the sparse matrix obtained from the `crqa()` function (Listing 2) and the RP generated here appear to be "flipped" versions of each other. This is the case because the RP obtained from `crqa()` orders the sequence of states on the vertical axis from bottom to top in reverse chronological order (e.g., state 1 is at the top and state 15 is at the bottom), which is the opposite of how we normally view numbers on a coordinate plane (i.e., state 1 is at the bottom, near 0, and state orders increase).

We use the `crqa()` function to extract the RPs and recurrence metrics of four more example dyads. Dyads 1 and 2 have similar frequencies of number talk utterances but differ on the recurrence metrics, while Dyads 3 and 4 have similar counts of number talk utterances and RR, but differ on the other recurrence metrics (Table 4). The RPs of these example dyads are generated using `ggplot2` (Figure 3). A brief discussion of the differences between these dyads' interactions is provided in the next section.

Interpreting RPs and CRQA measures

Visually, we observe that even dyads with similar frequencies of number talk vary widely in their temporal and dynamic structure. For instance, the extent to which dyads' number talk co-occurs (RR) differs, such that there is relatively more alignment (greater RR) in Dyad 1 than 2.

Listing 3 ■ Deriving the RP for the toy example using `ggplot2`

```
# extract RP from the crqa output
recurrence_matrix <- melt(as.matrix(example_crqa$RP),
                          varnames = c("toy_child", "toy_parent"))

recurrence_plot <- recurrence_matrix %>%
  ggplot(mapping = aes(x = toy_parent,
                       y = toy_child,
                       fill = value)
         ) +
  geom_raster() +
  scale_fill_manual(values=c("purple", # purple for recurrent points
                             "#F5F5F5"), # gray for non-recurrence points
                    breaks=c(TRUE, FALSE)) +
  geom_abline(intercept = 0, slope = 1) # add line of incidence

# combine data series into a data frame
plot_data <- cbind(toy_parent, toy_child) %>%
  as.data.frame() %>%
  rowid_to_column("utterance_order") %>%
  mutate(x_binary = ifelse(toy_parent == 1, utterance_order, NA), # NA to ensure no
    rug mark appears when number talk is *not* being used
         y_binary = ifelse(toy_child == 1, utterance_order, NA)
         ) %>%
  # add rug colors for each speaker
  mutate(rug_color = ifelse(
      !is.na(x_binary) & is.na(y_binary), "red", # child
      ifelse(!is.na(y_binary) & is.na(x_binary), "blue", NA) # parent
    )
  )

# add rugs to the axes
recurrence_plot <- recurrence_plot +
  geom_rug(inherit.aes = FALSE, # ensure that the mapping from above does not get
   combined
          data = plot_data,
          mapping = aes(x = x_binary,
                        y = y_binary,
                        color = rug_color)) +
  scale_color_manual(values=c("red", # red for child
                              "blue")) # blue for parent

# adjust RP aesthetics
recurrence_plot <- recurrence_plot +
  theme(axis.line = element_blank(), # remove axis lines/tick marks
        legend.position = "none") +  # remove legend
  coord_equal() + # make square
  labs(x = "Parent data series", y = "Child data series") # add axis labels

# show plot
recurrence_plot
```

Figure 2 ■ RP of the toy example using `ggplot2`

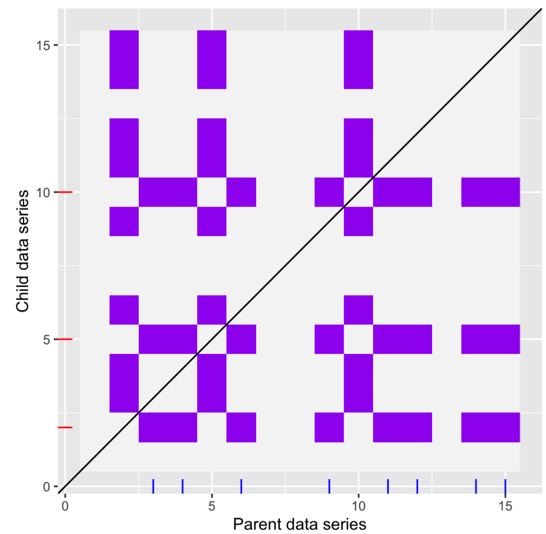


Table 4 ■ Utterance frequencies and recurrence metrics of the example dyads

Dyad	Total utterances	Child NT	Parent NT	Total NT	RR	DET	meanL	LAM	TT
1	312	53	39	92	3.16	24.79	2.16	40.99	2.84
2	322	19	67	86	2.46	37.86	2.12	23.17	2.48
3	264	17	38	55	1.85	40.87	2.13	18.42	2.80
4	282	38	17	55	1.62	27.24	2.10	61.69	2.61

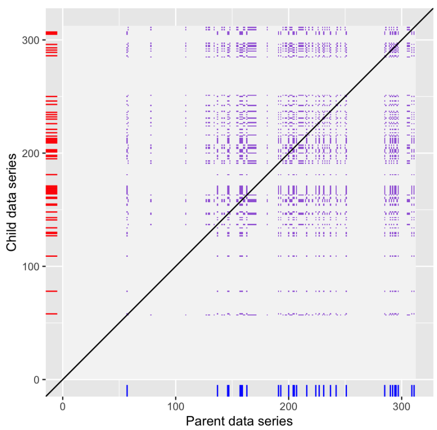
Note. Total utterances/NT = sum of both speakers' input. NT = number talk, RR = recurrence rate, DET = determinism, meanL = mean diagonal line, LAM = laminarity, TT = trapping time.

Also, it can be seen that the co-occurrence of number talk is distributed relatively evenly throughout Dyad 2's conversation, while the co-occurrence of talk in Dyad 1 occurs mostly in the middle to the end of their interaction. Dyad 2 exhibits greater exchange of number talk utterances (higher DET) than Dyad 1, and Dyad 1 used more consecutive number talk utterances (greater LAM) than Dyad 2. In other words, Dyad 2 engaged in relatively more "back and forth" interactions or turn-taking when discussing numbers than Dyad 1. In contrast, Dyad 1's conversation involved more consecutive number talk utterances, e.g., longer number talk responses, by one or both speakers than Dyad 2's conversation. An examination of Dyad 1's RP, specifically the blue (parent) and red (child) tick marks along the x- and y-axes, respectively, reveals that the child (more often than the parent) employed consecutive number utterances immediately after the parent used number talk. Thus, while Dyads 1 and 2 have similar frequencies of total number talk utterances, their temporal organization and patterns of number talk alignment differ.

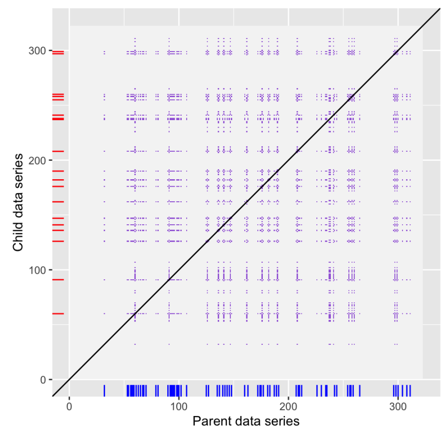
Additionally, Dyad 3 engaged in more (higher DET) and slightly longer (on average; higher meanL) number talk exchanges than Dyad 4. In other words, Dyad 3's turn-taking when discussing numbers generally lasted longer than Dyad 4's, particularly toward the end of Dyad 3's conversation which can be seen in their RP at the top right corner (starting slightly before the 200th utterance). In contrast, as observed in Dyad 4's RP, their number talk exchanges were shorter but occurred frequently in the beginning to middle of their conversation. Further, while Dyad 4 used more consecutive number talk utterances (greater LAM) than Dyad 3, on average, Dyad 3's consecutive number talk utterances were slightly longer (TT). Even though Dyads 3 and 4 are quantitatively similar in their alignment of number talk (RR) as well as their frequency of number talk utterances, their structures of alignment differ. Thus, CRQA can be used to derive measures that offer additional characterizations of dyadic conversations beyond traditional frequency measures like the quantity of utterances.

Figure 3 ■ Example Dyads' Recurrence Plots. Dyads 1 and 2 have similar frequencies of number talk utterances, but differ on the recurrence metrics. Dyads 3 and 4 have similar frequencies of number talk utterances and recurrence rates, but differ on the other recurrence metrics.

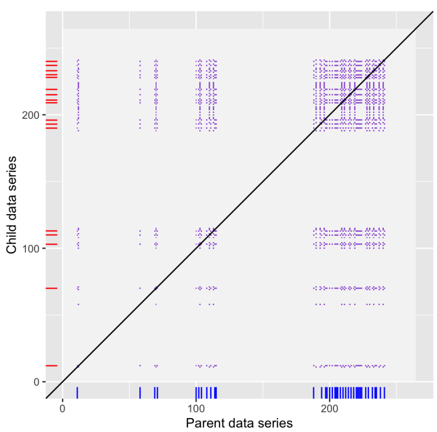
(a) Dyad 1



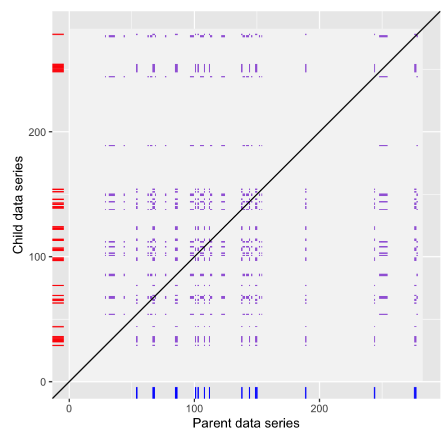
(b) Dyad 2



(c) Dyad 3



(d) Dyad 4



As mentioned previously, these CRQA metrics are purely descriptive. We can provide a richer description of an interaction by matching parts of the RP with the transcription and conducting, for example, conversation or discourse analysis. For instance, we may focus on a section of the RP with many diagonal line structures (e.g., top right corner of Dyad 3's RP in Figure 3), examine exactly what dyads are talking about, and then compare it to another part of the RP where we observe lengthy vertical line structures. Then, we may explore the relation between the observed interaction patterns or structures, the speech content, and the broader sociocultural or developmental context. If we want to draw inferences about dyadic interactions, we would need to collect data from many dyads and employ a variety of inferential statistical techniques. This might look like correlating the CRQA metrics with a variety of child outcomes (between-dyad differences) or examining differences in alignment by experimental condition(s) (between-group differences). The reader is encouraged to read our companion paper (Duong, Davis, et al., 2024) for an example of this approach, which examines how CRQA measures of alignment in parent-child number talk compare to frequencies of talk, and how these measures relate to children's math abilities.

Conclusions and future directions

The goal of this paper was to provide a simple tutorial of categorical cross-recurrence quantification analysis for obtaining more sensitive measures of dyadic interactions, specifically the extent to which dyads exhibit mutual alignment and their patterns of alignment. In a step-by-step tutorial in the R programming language, we demonstrated (1) the ease of applying CRQA to existing temporally ordered data and (2) that CRQA metrics offer descriptions of dyadic interactions beyond traditional frequency measures such as counts of utterances. This method can be used to explore our typical assumption that linguistic and social interactions are defined by the dynamic and reciprocal exchange of information and experiences.

Though our CRQA tutorial used categorical data with six unique states, this method can be used with numerical continuous data (see Coco & Dale, 2014), which is useful for studying states like movements. Also, as mentioned previously, other metrics that were not discussed in this paper, such as entropy, can be obtained from the crqa() function in R. Entropy describes the randomness of the diagonal line structures (e.g., number talk exchanges), which may be useful to researchers who are interested in the stability or regularity of systems (See the crqa package documentation at cran.r-project.org/web/packages/crqa/index.html for more information).

Further, CRQA can be used to explore "windowed" dynamics, which is achieved by partitioning the data series into a number of sub-series of a certain size. These sub-series can either overlap or not, and recurrence metrics are calculated for each sub-series to examine how cross-recurrence changes over time (Coco et al., 2021). CRQA can also be used to examine leader-follower dynamics (e.g., Coco et al., 2021; Dindar et al., 2020). One could quantify the recurrence of states between two series over different lags from the LOI and examine where the co-occurrence of states peaks.[3] If the "peak" of co-occurrences occurs along the LOI, this suggests that alignment is strongest between two data series at lag 0, e.g., when two actors are behaving at the same time. In contrast, if the "peak" occurs off the LOI, this suggests that the states of one series follows the other, e.g., one actor tends to follow the other. It is important to note that these descriptions of leader-follower dynamics do not suggest a causal story, i.e., that if Person A "lags" behind Person B, it is not necessarily the case that Person B caused Person A's behavior (Coco et al., 2021). Lastly, CRQA can be extended to examine the alignment between multidimensional state or time series, which are relevant to the study of behaviors such as eye, hand, and facial movements (e.g., smiling, raising eye brows) (Wallot & Leonardi, 2018). Overall, CRQA is a useful method for quantifying the patterns and extent of alignment in dyadic interactions. These aforementioned extensions can provide even more description and enrich our understanding of interactions beyond traditional frequency measures.

Authors' note

The data and tutorial code are available as supplementary material and can be accessed at https://github.com/s-duong/crqa-number-talk or as supplementary material on this journal's web site. The first author can also be contacted at shirleyduong5@gmail.com.

We are thankful to our project team for their assistance with data collection and transcription coding, our community partners that served as testing and recruitment sites, and the participating families from which the observational data are derived. This project was primarily funded by the Eunice Kennedy Shriver National Institute of Child Health and Human Development (1 R01 HD093689-01A1) to Heather J. Bachman, Melissa Libertus, and Elizabeth Votruba-Drzal. Additionally, a grant from the National Science Foundation (NSF; 1920545) helped fund the research team. Shirley Duong was supported by the NSF through their Graduate Research Fellowship. This project also benefitted from conversations about related studies funded by a Scholar Award from the James S. McDonnell Foundation to Melissa Libertus and an internal award from the Learning Research and Development Center at the University of Pittsburgh. We appreciate the editor and the anonymous reviewers for their comments. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the NICHD, the NSF, the LRDC, the James S. McDonnell Foundation, or the reviewers.

References

Afsar, O., Tirnakli, U., & Marwan, N. (2018). Recurrence quantification analysis at work: Quasi-periodicity based interpretation of gait force profiles for patients with parkinson disease. *Scientific Reports*, *8*(1), 9102–1. doi: 10.1038/s41598-018-27369-2.

Anderson, N. J., Graham, S. A., Prime, H., Jenkins, J. M., & Madigan, S. (2021). Linking quality and quantity of parental linguistic input to child language skills: A meta-analysis. *Child Development*, *92*(2), 484–501. doi: 10.1111/cdev.13508.

Angus, D. (2019). Recurrence methods for communication data, reflecting on 20 years of progress. *Frontiers in Applied Mathematics and Statistics*, *5*, 54. doi: 10.3389/fams.2019.00054.

Bachman, H. J., Elliott, L., Duong, S., Betancur, L., Navarro, M. G., Votruba-Drzal, E., & Libertus, M. (2020). Triangulating multi-method assessments of parental support for early math skills. *Frontiers in Education*, *5*, 589514. doi: 10.3389/feduc.2020.589514.

[3] Note that in our tutorial, we forced non-recurrence. This resulted in RPs that were symmetrical along the LOI. To examine follower-leader dynamics, we could have allowed speakers to have the same event codes at the same time points.

Bachman, H. J., Miller, P., Elliott, L., Duong, S., Libertus, M., & Votruba-Drzal, E. (2022). Associations among socioeconomic status, preschool-aged children's number skills, and spatial skills: The role of executive function. *Journal of Experimental Child Psychology*, *221*, 105453. doi: 10.1016/j.jecp.2022.105453.

Bakeman, R., & Quera, V. (2011). *Sequential analysis and observational methods for the behavioral sciences*. Cambridge University Press. doi: 10.1017/CBO9781139017343.

Bollenrücher, M., Darwiche, J., & Antonietti, J. (2023). Dyadic pattern analysis using longitudinal actor-partner interdependence model with Markov chains for unique case analysis. *The Quantitative Methods in Psychology*, *19*(3), 230–243. doi: 10.20982/tqmp.19.3.p230.

Chen, M., & Ferrer, E. (2022). Assessing dynamical associations in dyadic interactions across multiple time scales via a Bayesian hierarchical vector autoregressive model. *Journal of Social and Personal Relationships*, *40*(4), 1079–1102. doi: 10.1177/02654075221137865.

Coco, M. I., & Dale, R. (2014). Cross-recurrence quantification analysis of categorical and continuous time series: An r package. *Frontiers in Psychology*, *5*, 72510–1. doi: 10.3389/fpsyg.2014.00510.

Coco, M. I., Monster, D., Leonardi, G., Dale, R., & Wallot, S. (2021). Unidimensional and multidimensional methods for recurrence quantification analysis with crqa. *The R Journal*, *13*(1), 1–30.

Cox, R. F. A., & van Dijk, M. (2013). Microdevelopment in parent-child conversations: From global changes to flexibility. *Ecological Psychology*, *25*(3), 304–315. doi: 10.1080/10407413.2013.810095.

Curtin, P., Curtin, A., Austin, C., Gennings, C., Tammimies, K., Bölte, S., & Arora, M. (2017). Recurrence quantification analysis to characterize cyclical components of environmental elemental exposures during fetal and postnatal development. *PLOS ONE*, *12*(11), e0187049. doi: 10.1371/journal.pone.0187049.

Dale, R., & Spivey, M. J. (2005). Categorical recurrence analysis of child language. *Proceedings of the Annual Meeting of the Cognitive Science Society*, *27*, 530–535.

Dale, R., & Spivey, M. J. (2006). Unraveling the dyad: Using recurrence analysis to explore patterns of syntactic coordination between children and caregivers in conversation: Language learning. *Language Learning*, *56*(3), 391–430. doi: 10.1111/j.1467-9922.2006.00372.x.

Dale, R., Warlaumont, A. S., & Richardson, D. C. (2011). Nominal cross recurrence as a generalized lag sequential analysis for behavioral streams. *International Journal of Bifurcation and Chaos*, *21*(4), 1153–1161. doi: 10.1142/S0218127411028970.

Datavyu Team. (2014). *Datavyu: A video coding tool.* Retrieved June 24, 2024, from https://datavyu.org/

Davis, T. J. (2017). Modeling behavioral interaction as a nonlinear dynamical system cross-recurrence quantification analysis. In *Researching interactive communication behavior: A sourcebook of methods and measures* (pp. 261–272). SAGE.

Denby, J., & Yurovsky, D. (2019). Parents' linguistic alignment predicts children's language development. *Annual Meeting of the Cognitive Science Society*, 1627–1632.

Dindar, M., Jarvela, S., Ahola, S., Huang, X., & Zhao, G. (2020). Leaders and followers identified by emotional mimicry during collaborative learning: A facial expression recognition study on emotional valence. *IEE Transactions of Affective Computing*, *13*(3), 1390–1400. doi: 10.1109/TAFFC.2020.3003243.

Duong, S., Bachman, H. J., Votruba-Drzal, E., & Libertus, M. E. (2021). What's in a question? parents' question use in dyadic interactions and the relation to preschool-aged children's math abilities. *Journal of Experimental Child Psychology*, *211*, 105213. doi: 10.1016/j.jecp.2021.105213.

Duong, S., Davis, T., Bachman, H. J., Votruba-Drzal, E., & Libertus, M. E. (2024). Dynamic structures of parent-child number talk: An application of categorical cross-recurrence analysis and companion to duong et al., 2024. *The Quantitative Methods for Psychology*, *20*, 137–155. doi: 10.20982/tqmp.20.2.p137..

Eckmann, J.-P., Kamphorst, S. O., & Ruelle, D. (1987). Recurrence plots of dynamical systems. *Europhysics Letters (EPL)*, *4*(9), 973–977. doi: 10.1209/0295-5075/4/9/004.

Fox, D. S., Elliott, L., Bachman, H. J., Votruba-Drzal, E., & Libertus, M. E. (2024). Diversity of spatial activities and parents' spatial talk complexity predict preschoolers' gains in spatial skills. *Child Development*, *95*, 734–749. doi: 10.1111/cdev.14024.

Fusaroli, R., Konvalinka, I., & Wallot, S. (2014). Analyzing social interactions: The promises and challenges of using cross recurrence quantification analysis. In N. Marwan, M. Riley, A. Giuliani, & C. L. Webber (Eds.), *Translational recurrences* (pp. 137–155, Vol. 103). Springer International Publishing. doi: 10.1007/978-3-319-09531-8_9.

Fusaroli, R., Weed, E., Rocca, R., Fein, D., & Naigles, L. (2023). Caregiver linguistic alignment to autistic and typically developing children: A natural language processing approach illuminates the interactive components of language development. *Cognition*, *236*(105422). doi: 10.1016/j.cognition.2023.105422.

Gallotti, M., Fairhurst, M. T., & Frith, C. D. (2017). Alignment in social interactions. *Consciousness and Cognition*, *48*, 253–261. doi: 10.1016/j.concog.2016.12.002.

Gates, K. M., & Liu, S. (2016). Methods for quantifying patterns of dynamic interactions in dyads. *Assessment*, *23*(4), 459–471. doi: 10.1177/1073191116641508.

Hamaker, E. L., Zhang, Z., & Van der Maas, H. L. J. (2009). Using threshold autoregressive models to study dyadic interactions. *Psychometrika*, *74*(4), 727–745. doi: 10.1007/s11336-009-9113-4.

He, J., Shang, P., & Zhang, Y. (2020). Global recurrence quantification analysis and its application in financial time series. *Nonlinear Dynamics*, *100*(1), 803–829. doi: 10.1007/s11071-020-05543-4.

Heunis, T., Aldrich, C., Peters, J. M., Jeste, S. S., Sahin, M., Scheffer, C., & de Vries, P. J. (2018). Recurrence quantification analysis of resting state eeg signals in autism spectrum disorder – a systematic methodological exploration of technical and demographic confounders in the search for biomarkers. *BMC Medicine*, *16*(101), 1–17. doi: 10.1186/s12916-018-1086-7.

Koenker, R., & Ng, P. (2003). Sparsem : A sparse matrix package for r*. *Journal of Statistical Software*, *8*(6), 1–9. doi: 10.18637/jss.v008.i06.

Leclère, C., Viaux, S., Avril, M., Achard, C., Chetouani, M., Missonnier, S., & Cohen, D. (2014). Why synchrony matters during mother-child interactions: A systematic review. *PLoS One*, *9*(12), e113571. doi: 10.1371/journal.pone.0113571.

Lira-Palma, D., González-Rosales, K., Castillo, R. D., Spencer, R., & Fresno, A. (2018). Categorical cross-recurrence quantification analysis applied to communicative interaction during ainsworth's strange situation. *Complexity*, *2018*, 1–15. doi: 10.1155/2018/4547029.

Main, A., Paxton, A., & Dale, R. (2016). An exploratory analysis of emotion dynamics between mothers and adolescents during conflict discussions. *Emotion*, *16*(6), 913–928. doi: 10.1037/emo0000180.

McCormick, E. N., & Blaha, L. M. (2021). Practical interpretation and insights with recurrence quantification analysis for decision making research. *Proceedings of the Annual Meeting of the Cognitive Science Society*, *43*, 13–14. https://escholarship.org/uc/item/39q433xv

Menninga, A., van Dijk, M., Steenbeek, H., & van Geert, P. (2017). Language use in real-time interactions during early elementary science lessons: The bidirectional dynamics of the language complexity of teachers and students: Language use in real-time early science lessons. *Language Learning*, *67*(2), 284–320. doi: 10.1111/lang.12219.

Misiek, T., Favre, B., & Fourtassi, A. (2020). Development of multi-level linguistic alignment in child-adult conversations. *Proceedings of the Workshop on Cognitive Modeling and Computational Linguistics*, 54–58. doi: 10.18653/v1/2020.cmcl-1.7.

Pan, B. A., Rowe, M. L., Spier, E., & Tamis-Lemonda, C. (2004). Measuring productive vocabulary of toddlers in low-income families: Concurrent and predictive validity of three sources of data. *Journal of Child Language*, *31*(3), 587–608. doi: 10.1017/S0305000904006270.

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in head start families. *Cognitive Development*, *35*, 15–33. doi: 10.1016/j.cogdev.2014.11.002.

Recurrence plots and their quantifications: Expanding horizons. (2016). In C. L. Webber, C. Ioana, & N. Marwan (Eds.), *Proceedings of the 6th international symposium on recurrence plots*. Springer International Publishing. doi: 10.1007/978-3-319-29922-8.

Richardson, D. C., & Dale, R. (2005). Looking to understand: The coupling between speakers' and listeners' eye movements and its relationship to discourse comprehension. *Cognitive Science*, *29*(6), 1045–1060. doi: 10.1207/s15516709cog0000_29.

Rohlfing, K. J., Leonardi, G., Nomikou, I., Raczaszek-Leonardi, J., & Hullermeier, E. (2020). Multimodal turn-taking: Motivations, methodological challenges, and novel approaches. *IEEE Transactions on Cognitive and Developmental Systems*, *12*(2), 260–271. doi: 10.1109/TCDS.2019.2892991.

Solomon, D. H., Brinberg, M., Bodie, G. D., Jones, S., & Ram, N. (2021). A dynamic dyadic systems approach to interpersonal communication. *Journal of Communication*, *71*(6), 1001–1026. doi: 10.1093/joc/jqab035.

Takens, F. (1981). Detecting strange attractors in turbulence. *Lecture Notes in Mathematics*, *898*, 366–381.

van Dijk, M., van Geert, P., Korecky-Kröll, K., Maillochon, I., Laaha, S., Dressler, W. U., & Bassano, D. (2013). Dynamic adaptation in child-adult language interaction: Dynamic adaptation in child-adult language interaction. *Language Learning*, *63*(2), 243–270. doi: 10.1111/lang.12002.

Wallot, S., & Leonardi, G. (2018). Analyzing multivariate dynamics using cross-recurrence quantification analysis (crqa), diagonal-cross-recurrence profiles (dcrp), and multidimensional recurrence quantification analysis (mdrqa) – a tutorial in r. *Frontiers in Psychology*, *9*. doi: 10.3389/fpsyg.2018.02232.

Webber, C. L., & Zbilut, J. P. (1994). Dynamical assessment of physiological systems and states using recurrence plot strategies. *Journal of Applied Physiology*, *76*(2), 965–973. doi: 10.1152/jappl.1994.76.2.965.

Webber, C. L., & Zbilut, J. P. (2007). Recurrence quantifications: Feature extractions from recurrence plots. *International Journal of Bifurcation and Chaos*, *17*(10), 3467–2475. doi: 10.1142/S0218127407019226.

Wickham, H. (2007). Reshaping data with the reshape package. *Journal of Statistical Software*, *21*(12), 1–99. https://www.jstatsoft.org/v21/i12/

Wickham, H. (2016). *Ggplot2: Elegant graphics for data analysis* [Springer-Verlag New York. -4]. doi: 10.1007/978-3-319-24277-4.

Xu, T. L., & Yu, C. (2016, August). *Quantifying joint activities using cross-recurrence block representation*. Cognitive Science Society.

Yurovsky, D., Doyle, G., & Frank, M. C. (2016). Linguistic input is tuned to children's developmental level. *Annula Meeting of the Cognitive Science Society*, 2093–2098.

Zbilut, J. P., & Webber, C. L. (1992). Embeddings and delays as derived from quantification of recurrence plots. *Physics Letters A*, *171*(3-4), 199–203. doi: 10.1016/0375-9601(92)90426-M.

Open practices

The *Open Material* badge was earned because supplementary material(s) are available on the journal's web site.

Citation

Duong, S., Davis, T. J., Bachman, H. J., Votruba-Drzal, E., & Libertus, M. E. (2024). Exploring dynamic structures of dyadic conversations using categorical cross recurrence quantification analysis: A tutorial. *The Quantitative Methods for Psychology*, *20*(2), 121–136. doi: 10.20982/tqmp.20.2.p121.

Received: 15/01/2023 ~ Accepted: 28/05/2024